Time Series Exercise Sheet 1

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A note on notation: sometimes it is useful to discuss the autocovariance sequence of multiple different time series. Often, for a time series $\{X_t\}$, the autocovariance will be written as $\gamma_X(\tau)$, even if it is really a sequence. This is because the notation is usually cleaner than the alternatives such as $\gamma_{X,\tau}$. Whether the autocovariance is a sequence or a function will be clear from context.

Exercise 1.1

Let X_t be distributed as a student t distribution on one degree of freedom independently across t. Is $\{X_t\}$ weakly stationary?

Exercise 1.2

Let $\{X_t\}$ be second order stationary with autocovariance γ_τ and ε_t be iid with finite variance. Furthermore, assume $\{X_t\}$ and $\{\varepsilon_t\}$ are mutually independent. Determine the autocovariance of $Z_t = X_t + \varepsilon_t$.

Exercise 1.3

Let Y_t be iid Gaussian random variables of mean 0 and variance σ_Y^2. Let a, b and c be constants. Which of the following processes are weakly stationary/strongly stationary, and if so, give their mean and ACVS.

1. $X_t = a + bY_t + cY_{t-1}$,
2. $X_t = a + bY_0$,
3. $X_t = Y_1 \cos(ct) + Y_2 \sin(ct)$
4. $X_t = Y_0 \cos(ct)$

Exercise 1.4

Determine the autocovariance of ϵ_t which is uncorrelated across t, but with fixed variance σ^2 and mean zero.

Exercise 1.5

Determine the autocovariance of $X_t = \theta_1\epsilon_t + \theta_2\epsilon_{t-1}$ where ϵ_t which is mean zero, uncorrelated across t but with fixed variance σ^2.

Exercise 1.6

Suppose that $\{X_t\}$ and $\{Y_t\}$ are uncorrelated stationary time series, i.e. X_t and Y_s are uncorrelated for every choice of t and s. Show that the sequence $\{X_t + Y_t\}$ is stationary with an autocovariance that is the sum of the autocovariance sequences of $\{X_t\}$ and $\{Y_t\}$.

Exercise 1.7

Suppose that $\{X_{t,1}\}, \{X_{t,2}\}, \ldots, \{X_{t,m}\}$ are stationary processes with zero means and

$$\gamma_j(\tau) = \mathrm{Cov}\left(X_{t,j}, X_{t+\tau,j}\right).$$

If $\mathbb{E}\left[X_{t,j}X_{t+\tau,k}\right] = 0$ for all t, τ and $j \neq k$ determine the autocovariance sequence of

$$X_t = \sum_{j=1}^{m} X_{t,j}.$$